

CMI Mathematics Colloquium

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Minimal elements of Shi regions in affine Weyl groups

Balthazar Charles

Consider the union of hyperplanes orthogonal to the roots of an affine Weyl group –its Coxeter arrangement– and keep only the hyperplanes associated with roots of height 0 or 1 –the Shi arrangement. Among many good properties of the Shi arrangement, we are interested in the fact that any of its regions contains an alcove of the Coxeter arrangement that is, in some sense, minimal. This alcove can be encoded by an integer vector –its Shi vector– satisfying some relations. How can we, given a Shi region, compute the Shi vector of its minimal element ?

In this talk, we present a bijection between parking functions in the sense of [Armstrong, Reiner, Rhoades '15] and the Shi vectors of the minimal elements of Shi regions in affine Weyl groups. To that end, we study the structure of the Shi relations on the coefficients of Shi vectors. We discuss the specialization of our result in the case of classical Weyl groups. Time allowing, we may discuss a proof of [Dyer, Hohlweg '16, Conjecture 2] as an application of this description we provide.