CMI Mathematics Colloquium

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Symplectic integrals on the moduli space of rank two vector bundles

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Given a Riemann surface Σ, the moduli space $\mathcal{M}$ of rank two vector bundles with trivial determinant is, by the Narasimhan-Seshadri Theorem, in bijection with the space of (equivalence classes of) representations in $SU(2)$ of the fundamental group of the surface Σ.

In the latter avatar, the space $\mathcal{M}$ has a symplectic structure and a corresponding finite measure, the Liouville measure. Normalised to total mass one this gives a probability measure. There is a natural class of functions $W_C : \mathcal{M} \to [0, 1]$ parametrised by isotopy classes of loops $C \subset \Sigma$. These are called Wilson loop functions by physicists and Goldman functions by mathematicians. With applications in mind, I present a simple scheme to compute joint distributions of these functions for families of loops. This is possible because of a miracle of symplectic geometry called the Duistermaat-Heckman formalism (whose applicability in this context is due to L. Jeffrey and J. Weitsman) and a continuous analogue of the Verlinde algebra. The Verlinde algebra is well-known to physicists, representation theorists and algebraic geometers in the context of "generalised theta functions".